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IMPULSE SPECTRAL INTENSITY — WHAT IS IT?

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U.S. DEPARTMENT OF COMMERCE, Frederick B. Dent, Secretary

NATIONAL BUREAU OF STANDARDS, Richard W. Roberts, Director

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FOREWORD

The following report is the text of a paper presented at the Electromagnetic Compatibility Colloquium, U.S. Army Electronics Command, Ft. Monmouth, New Jersey, May 1-2, 1974. Sufficient interest was expressed by attendees at the colloquium to warrant making the text available in printed form. In the interest of economy and prompt response to requests for copies, the text has been kept in substantially the same form as was used in the oral presentation.

IMPULSE SPECTRAL INTENSITY -- WHAT IS IT?

ABSTRACT

The term, impulse spectral intensity, is often used in discussions concerning electromagnetic interference and broadband signal processing. In these discussions, the term is not used in a consistent manner, resulting in confusion, equivocation, and sometimes, errors. As a step towards improving this situation, the mathematical basis for spectral intensity is reviewed and certain of its features are clarified. Also, misuses of spectral intensity are discussed, along with its limitations and proper use.

Key words: Electromagnetic interference; Fourier transform; impulse spectral intensity; spectral intensity; spectrum amplitude; spectrum amplitude density.

1. INTRODUCTION

In EMC work, and in the EMC literature, we often encounter an electromagnetic quantity that has the dimensional units of volts per hertz, or more commonly, microvolts per megahertz. But as we engage in EMC work or read the literature, we sometimes are puzzled or confused by the variety of names that this quantity is given, or we may be suspicious that the way in which this quantity is being used is somehow different from the way that someone else has used it.

In this paper, I would like to talk about the quantity that has the dimensional units of volts per hertz with the purpose hopefully of clarifying what some of its important characteristics are, and possibly even helping to reduce some of the large amount of confusion, half truths, and whole untruths about it.

The most official name given to this quantity is found in the IEEE Dictionary [1]. That name is Spectrum Amplitude. Spectrum amplitude is defined there as $1/\pi$ times the <u>magnitude</u> of the Fourier transform of a time-domain signal function. This $1/\pi$ is an error, and should be replaced by the numeric 2, thus:

$$S(f) = 2|V(f)| \tag{1}$$

where

$$V(f) = \int_{-\infty}^{\infty} v(t)e^{-j2\pi ft}\,dt \tag{2}$$

and $S(f)$ = Spectrum Amplitude [2]. Other names that can be found in the EMC literature are Impulse Spectral Intensity, Spectral Intensity, Spectral Density, Voltage Spectrum, Impulse Strength and Interference Intensity. All of these names have been applied to quantities that have dimensional units of volts per hertz or its mathematical equivalent, <u>volt-seconds</u>.

I personally am not satisfied with any of these names, but I do agree with the definition as given in the proposed new IEEE standard on impulse strength [2]. Perhaps there are others of you who have wondered what is the theoretical or engineering basis for this quantity. In studying this matter, the most basic starting point that I have found is the Fourier transform of the time-domain signal. Therefore, I want to discuss some of the important characteristics of this Fourier transform as they pertain to the question, "Impulse Spectral Intensity -- What Is It?".

Before proceeding further, let me say what my major points will be.

FIRST, the Fourier Transform, in the general case, is mathematically a complex quantity. Its magnitude alone may not be adequate to meet a given need. Phase information may also be needed.

SECOND, the Fourier transform is an amplitude and phase distribution function, and not simply an amplitude.

THIRD, the Fourier transform is a continuously defined function of frequency, and not a discontinuous, discrete function of frequency.

FINALLY, the magnitude of the Fourier transform appears explicitly in many equations dealing with a signal function. This leads to the identification of a spectral amplitude quantity having engineering importance to EMC work.

Having made these statements, let me show you some justifications for and implications of these statements. Although I start out by focusing on the Fourier transform, I'll soon shift over to a type of spectral amplitude quantity.

2. FUNDAMENTALS

Every physical signal can, at least in principle if not in practice, be represented mathematically in both the time domain and the frequency domain. An algebraic equation representing the physical signal in the time domain can be mathematically transformed by the Fourier transform [3], [4] to represent the physical signal in the frequency domain. Conversely, the algebraic frequency-domain representation of the physical signal can be transformed by the inverse Fourier transform to produce an algebraic time-domain representation of the physical signal. The two equations defining these two transformations are as follows:

$$V(f) = \int_{-\infty}^{\infty} v(t)e^{-j2\pi ft}\, dt \tag{2}$$

$$v(t) = \int_{-\infty}^{\infty} V(f)e^{j2\pi ft}\, df \tag{3}$$

They are called the transform pair for the physical signal which is represented by v(t) in the time domain and by V(f) in the frequency domain. Equation (2) transforms the time-domain quantity into a frequency-domain representation; eq. (3) transforms the frequency-domain quantity into a time-domain representation. Both equations represent the same physical signal, and are simply two different ways of mathematical representation.

As an example, consider the single rectangular baseband pulse shown in figure 1. This signal has an amplitude of A from $t = 0$ to $t = \tau$, and is zero elsewhere. Although I have selected an idealized pulse-like signal for illustration, all of the observations and conclusions which we will discuss generally apply to any physical signal, regardless of whether it is a single pulse of arbitrary shape, or a series of pulses repeated at perfectly regular intervals, or a series of pulses repeated at irregular intervals, or continuous type signal funtions of regular or irregular shape. The Fourier transform of our example is $V_p(f)$ as given by the following equations (see Appendix A).

$$V_p(f) = \frac{A}{2\pi f}(\sin 2\pi f\tau - j2 \sin^2 \pi f\tau) \qquad (4)$$

$$= A\tau \left| \frac{\sin \pi f\tau}{\pi f\tau} \right| e^{-j\pi f\tau} \qquad (5)$$

Equation (4) shows that $V_p(f)$ is a complex quantity, having both a real and an imaginary part. This is true in general of any V(f), although some signal representations, depending upon their waveform and the choice of time origin, may have only a real or an imaginary part.

Equation (5) is written in terms of the magnitude,

$$|V_p(f)| = A\tau \left| \frac{\sin \pi f\tau}{\pi f\tau} \right| \qquad (6)$$

and phase,

$$\phi_p(f) = -\pi f\tau \qquad (7)$$

of the complex frequency-domain signal representation. To repeat, the Fourier transform is a complex quantity having both a magnitude and phase.

A second characteristic of the Fourier transform, V(f) is that it is the integral over time of a time function (see eq. (2)). This means that V(f) is geometrically an AREA. In our example,

$$V_p(f) = \int_{-\infty}^{\infty} v_p(t)e^{-j2\pi ft}\, dt \tag{8}$$

$$= \int_{0}^{\tau} Ae^{-j2\pi ft}\, dt \tag{9}$$

$$= \int_{-\infty}^{\infty} y\, dt \tag{10}$$

where

$$\begin{aligned} y &= Ae^{-j2\pi ft}, \quad 0 < t < \tau \\ &= 0 \text{ elsewhere.} \end{aligned} \tag{11}$$

This area is the area under the curve of eq. (11). The function y is a function of both frequency and time, and is complex. The real part of y in our example is

$$\mathrm{Re}[y] = A \cos 2\pi ft \tag{12}$$

and the imaginary part is

$$\mathrm{Im}[y] = -A \sin 2\pi ft. \tag{13}$$

Figure 2 shows a portion of a three-dimensional plot of the real part of y. It is three dimensional because y is a function of both frequency and time. The result is a wavy surface with a cosinusoidal variation in both directions. The real part of the Fourier transform, $\mathrm{Re}[V_p(f)]$, at a specified frequency, f, is the area under the cosine curve at frequency f in the time direction. For example, figure 3 shows four "slices" through this wavy surface parallel to the time-axis and perpendicular to the frequency-axis. The value of $\mathrm{Re}[V_p(f)]$ at frequency f_c is the area marked "c." The imaginary part of $V_p(f)$ can be found in a similar way, and then $V_p(f)$ is the vector sum of the real and the imaginary parts (see the vector diagram at frequency f_b).

Being an area, V(f) has dimensional units which are the product of the units of its constituent parts, height and base. If v(t) is a voltage, then V(f) has the units of volt-seconds. From this we can see that V(f) is not the amplitude per se of sinusoidal signals that are components of the physical signal in the frequency domain. These amplitudes are zero and would have the dimensional units of volts. Rather, V(f) is a measure of the amplitude distribution of a continuously defined frequency spectrum. It tells relative amplitudes as they are distributed with frequency. Thus it is a distribution function, a type of density function. Because of this, V(f) is often expressed in units of volts per hertz, which are dimensionally equivalent to the units of volt-seconds.

There is a strong potential hazard in choosing the units of volts per hertz rather than volt-seconds. It's true that mathematically they are equivalent. From a physical standpoint, volts per hertz suggests a density function whereas volt-seconds does not. And I can find no engineering justification for using one set of units in preference to the other. However, the units, volts per hertz, can mislead one into thinking that he can simply integrate V(f) over a finite interval of frequency and obtain a voltage that has some useful physical meaning. Such may not be the case. What does have useful meaning, as we have seen in eq. (3), is the integral of the product of V(f) with an external exponential function, which, for the infinite integral, yields the time-domain signal representation v(t). Incidentally, the infinite integral of just V(f) over the entire frequency interval is the value of the time-domain signal function at time t = 0. This is a basic theorem of Fourier transforms. Thus the integral of V(f) over a finite interval yields the part of v(t) at t = 0 contributed by the frequency-domain signal in that band of frequencies. To repeat, V(f) is a spectral distribution function.

Another characteristic of V(f) is that it is a continuously defined function of frequency. Let me show you what I mean. A plot of $|V_p(f)|$ and $\phi_p(f)$ for our example is given in figure 4. Although there may be points where V(f) is zero, it is generally true that there are no places where V(f) does not exist. In this way the Fourier transform representation of a physical signal is different from the Fourier Series representation [5] in which the spectrum is a discrete spectrum, and the spectral components are amplitudes, expressed in volts, of sinusoids at discrete frequencies. Having heard this, you may wonder why the spectrum of a so-called "periodic" pulse train is experimentally observed to be a line spectrum. The truth is, it is not truly a line spectrum, but only appears to be so because of the deficiencies of our test equipment. The Fourier Transform of a regularly repeating function, such as the one illustrated in figure 5(a), which is of finite duration T, as all physical signals are, is a sum of terms given by the equation

$$V_r(f) = A\tau f_o T \left\{ \sum_{n=-\infty}^{n=\infty} \frac{\sin \pi n f_o \tau}{\pi n f_o \tau} \cdot \frac{\sin \pi (f - n f_o) T}{\pi (f - n f_o) T} \right\} \tag{14}$$

where f_o is the repetition rate of the physical signal (see Appendix B). Each term of this sum can be plotted as an individual spectral amplitude distribution centered at a frequency $f = nf_o$. Three such terms are shown in figure 5(b) for n = 0 and n = ± 1. As the time interval T becomes longer and longer, the amplitude distributions become more and more compact in the frequency dimension, and, as T approaches infinity, the total amplitude distribution looks like a line spectrum with vast regions of zero amplitude

between lines. However, in these regions between the frequencies where the amplitude distribution is bunched up in clumps, $V_r(f)$ is not zero but only infinitesimally small. Because of this limiting form, which is what is observed in practice, many authors give the erroneous impression that the spectrum amplitude for a finite periodic signal is a line spectrum. And then, through confusion with the Fourier series line spectrum, this can mislead one into thinking that these lines represent the amplitudes of sinusoids at specific frequencies. Such is not the case when we are dealing with the Fourier transform. The ordinate values still have units of volt-seconds and are not amplitudes in volts.

3. APPLICATIONS OF V(f)

Thus far, we have talked about the nature of V(f). Now let us look at how V(f) is used, and this will lead us to identify a spectral amplitude.

One basic mathematical use of V(f) is to obtain the time-domain equation of the physical signal by taking the inverse Fourier transform of V(f). Here we can write the inverse transform equation in a form that shows the amplitude and phase of V(f). For our example, $v_p(t)$ is given by the equation (see Appendix C):

$$v_p(t) = A\tau \int_{-\infty}^{\infty} \left| \frac{\sin \pi f\tau}{\pi f\tau} \right| \cos 2\pi f(t-\tau/2)df. \tag{15}$$

Again we can see that the value of v(t) at any time t is an intergral over frequency of a function of frequency. Thus, v(t) is an AREA which, in our example, has the dimensional units of volts.

Figure 6 shows a portion of a three-dimensional plot of the integrand, z(f,t), of eq. (15), where

$$z(f,t) = A\tau \left| \frac{\sin \pi ft}{\pi ft} \right| \cos 2\pi f(t-\tau/2). \tag{16}$$

This wavy surface has a "sine x over x" shape in the frequency direction at time $t = \tau/2$ and a cosine shape in the time direction. The surface extends to plus and minus infinity in both the time and frequency domains. The area under the function z (f,t), at a specified time t, is the value of $v_p(t)$ at that time (see fig. 7).

Because v(t) for every physical real-world signal is a purely real function of time, V(f) is Hermitian, mathematically speaking, which simply means that its real part is always an even function, its imaginary part is always odd, and

$$V(f) = V^*(-f). \tag{17}$$

It is this mathematical property that allows us, and this is important in a practical sense, to write

$$v(t) = 2 \int_0^\infty |V(f)| \cos[2\pi ft + \phi(t)]df \tag{18}$$

for ALL real-world signals, not just for our example. This form of mathematical representation has the engineering virtue that the integral is over the positive frequency domain only.

The spectrum amplitude

$$S(f) = 2|V(f)| \tag{1}$$

thus appears explicitly in eq. (18).

The quantity $2|V(f)|$ also appears in most other working equations where a spectrum amplitude is involved, some of which I will show in a moment. I believe, therefore, that this quantity has utility as a measurand of the physical signal, and should be adopted by the EMC community as the quantity which is identified by the units of volts per hertz. I would suggest that it be named Spectrum Amplitude Density, or perhaps Amplitude Density. The name, spectrum amplitude density, would be in keeping with the frequency-domain characteristics that we have discussed. But the name is a matter for standards committee to decide.

S(f) appears in other applications, some of which are as follows:

1. If v(t) is applied to a two-port network which has an impulse response h(t), the response, r(t), is given by the equation

$$r(t) = \int_0^\infty S(f) \cdot |H(f)| \cdot \cos[2\pi ft + \phi(f) + \theta(f)]df \tag{19}$$

where $|H(f)|$ and $\theta(f)$ are the magnitude and phase of the transfer function of the network. It is this application for which the knowledge of spectrum amplitude has its greatest practical application, namely, for the calibration of Field Intensity Meters and Noise Meters.

2. Another application is the measurement of the impulse bandwidth, IB, of an idealized bandpass filter [2] using the equation

$$IB = \frac{R_{(max)}}{S(f)G} \tag{20}$$

where $R_{(max)}$ is the maximum value of the envelope of the network response r(t), and G is the network maximum gain.

3. The total energy per unit resistance, E_t, contained in the physical signal is given by the equations

$$E_t = \int_{-\infty}^{\infty} v^2(t)\,dt = \int_{-\infty}^{\infty} |V(f)|^2\,df \tag{21}$$

$$= \frac{1}{2}\int_{0}^{\infty} S^2(f)\,df. \tag{22}$$

The part, E_b, of E_t, that lies in the frequency band from f_1 to f_2 is given by the equation

$$E_b = \frac{1}{2}\int_{f_1}^{f_2} S^2(f)\,df. \tag{23}$$

The total energy per unit resistance, E_n, contained in the output signal from a two-port network is given by the equation

$$E_n = \frac{1}{2}\int_{0}^{\infty} S^2(f) \cdot |H(f)|^2\,df. \tag{24}$$

Finally, if we were to define a root mean square spectrum amplitude, S_{rms} [2], as

$$S_{rms} = \frac{S(f)}{\sqrt{2}} = \sqrt{2}|V(f)|, \tag{25}$$

then the square of S_{rms} is $2|V(f)|^2$, and the last three energy equations become

$$E_t = \int_{0}^{\infty} S^2_{rms}\,df \tag{26}$$

$$E_b = \int_{f_1}^{f_2} S^2_{rms}\,df \tag{27}$$

$$E_n = \int_{0}^{\infty} S^2_{rms} \cdot |H(f)|^2\,df. \tag{28}$$

4. SUMMARY

In summary, we have seen that:

First, the frequency-domain representation of a physical signal is a complex <u>mathematical</u> quantity with <u>both</u> a magnitude <u>and</u> a phase.

Second, this complex quantity, the Fourier transform, is an <u>amplitude distribution function</u> -- a type of <u>density</u> function -- and is not simply the amplitudes of individual spectral components. It must be treated as a density function and not as an amplitude.

Third, the Fourier transform of <u>any</u> <u>and</u> <u>every</u> physical signal is <u>continuously</u> defined throughout the <u>entire</u> <u>frequency</u> <u>domain</u>. It is <u>not</u> a <u>discrete</u> or <u>line</u> <u>function</u> of frequency.

Fourth, twice the <u>magnitude</u> of this Fourier transform is a useful engineering quantity because it appears in this form in many useful algebraic relationships. However, one may also need to know the phase of V(f).

Fifth, we should give this quantity a descriptive name, such as <u>Spectrum</u> <u>Amplitude</u> <u>Density</u>, and should agree upon one definition to the exclusion of all others. Such a definition should be in terms of signal parameters and not in terms of measurement network parameters as is found in parts of the literature.

Sixth, the preferable dimensional units of spectrum amplitude may be volt-seconds, although there are valid arguments for both volt-seconds and volts per hertz.

Finally, a root-mean-square spectrum amplitude density can be defined which may have utility for engineering purposes, especially when dealing with signal <u>power</u>.

Although I have limited this review to a few of the basic matters, I hope it has been helpful in providing the basis for a better understanding of spectrum amplitude, and for a more careful use of it in the future.

5. REFERENCES

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[6] Lathi, ibid., p. 130.

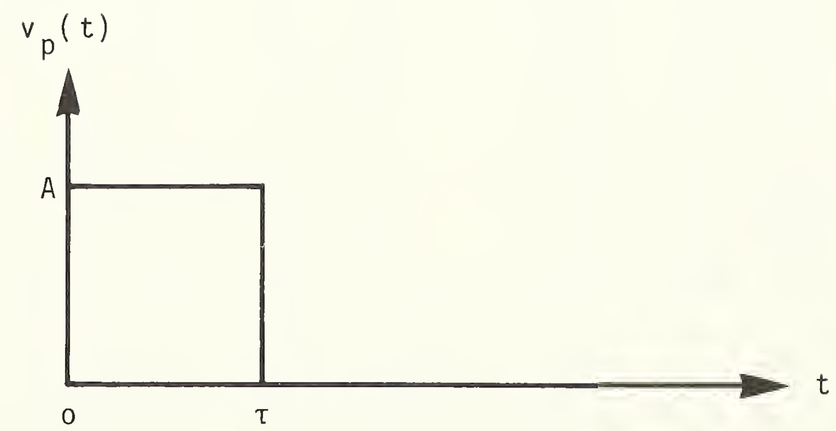

Fig. 1. Single rectangular baseband pulse.

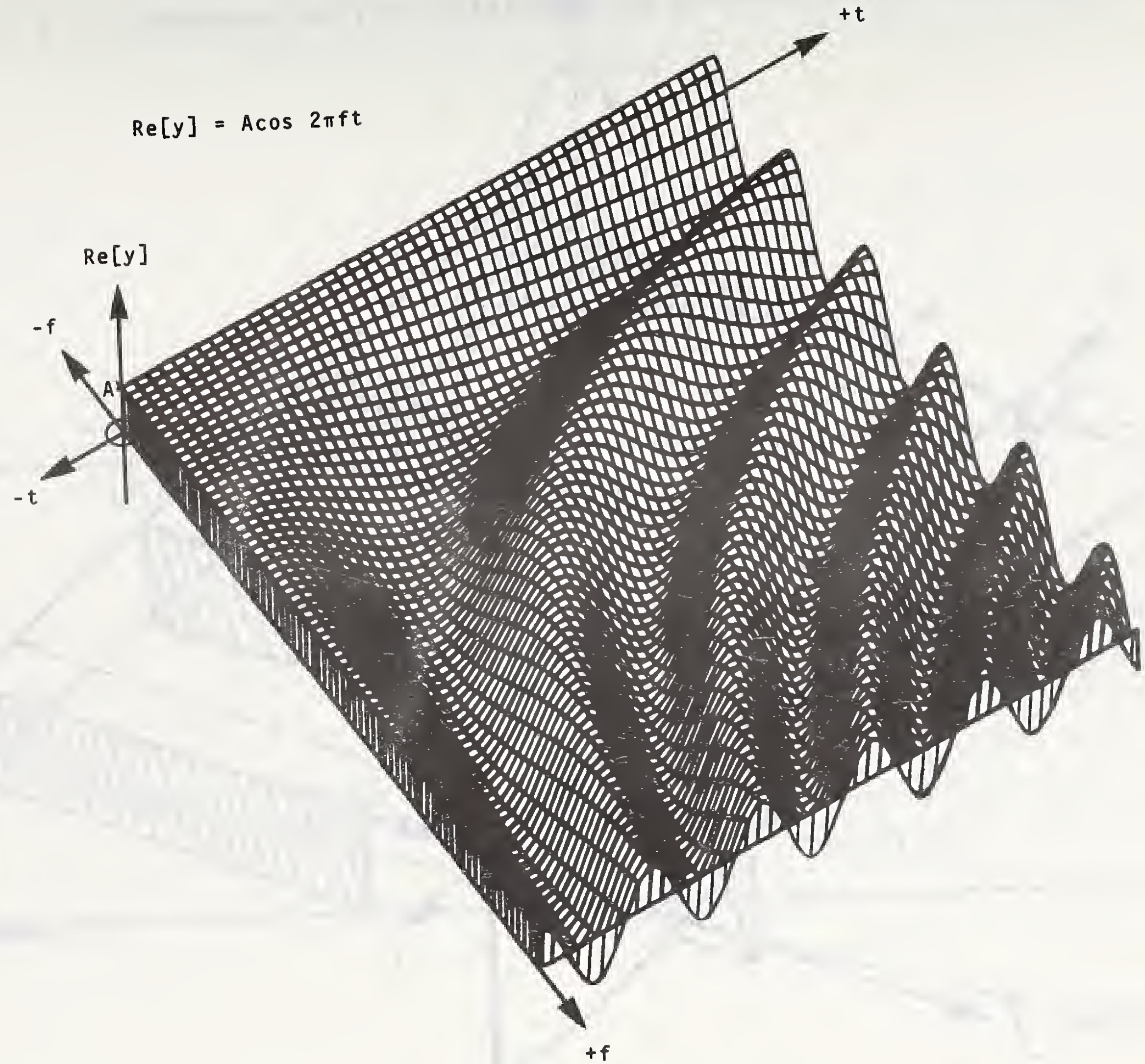

Fig. 2. Three-dimensional plot of $Re[y] = A \cos 2\pi ft$.

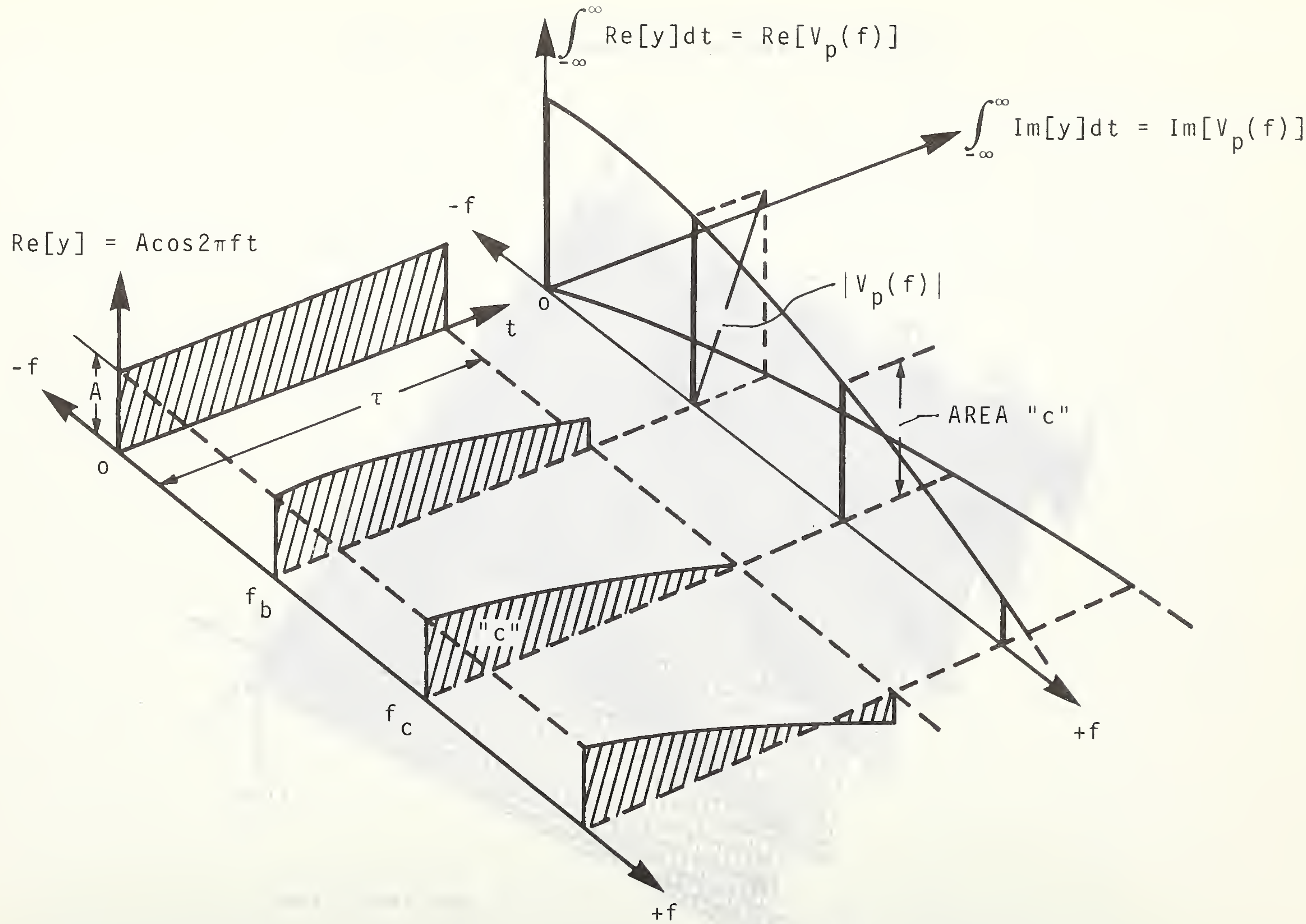

Fig. 3. Illustrating the relationship between Re[y] and Re[V_p(f)].

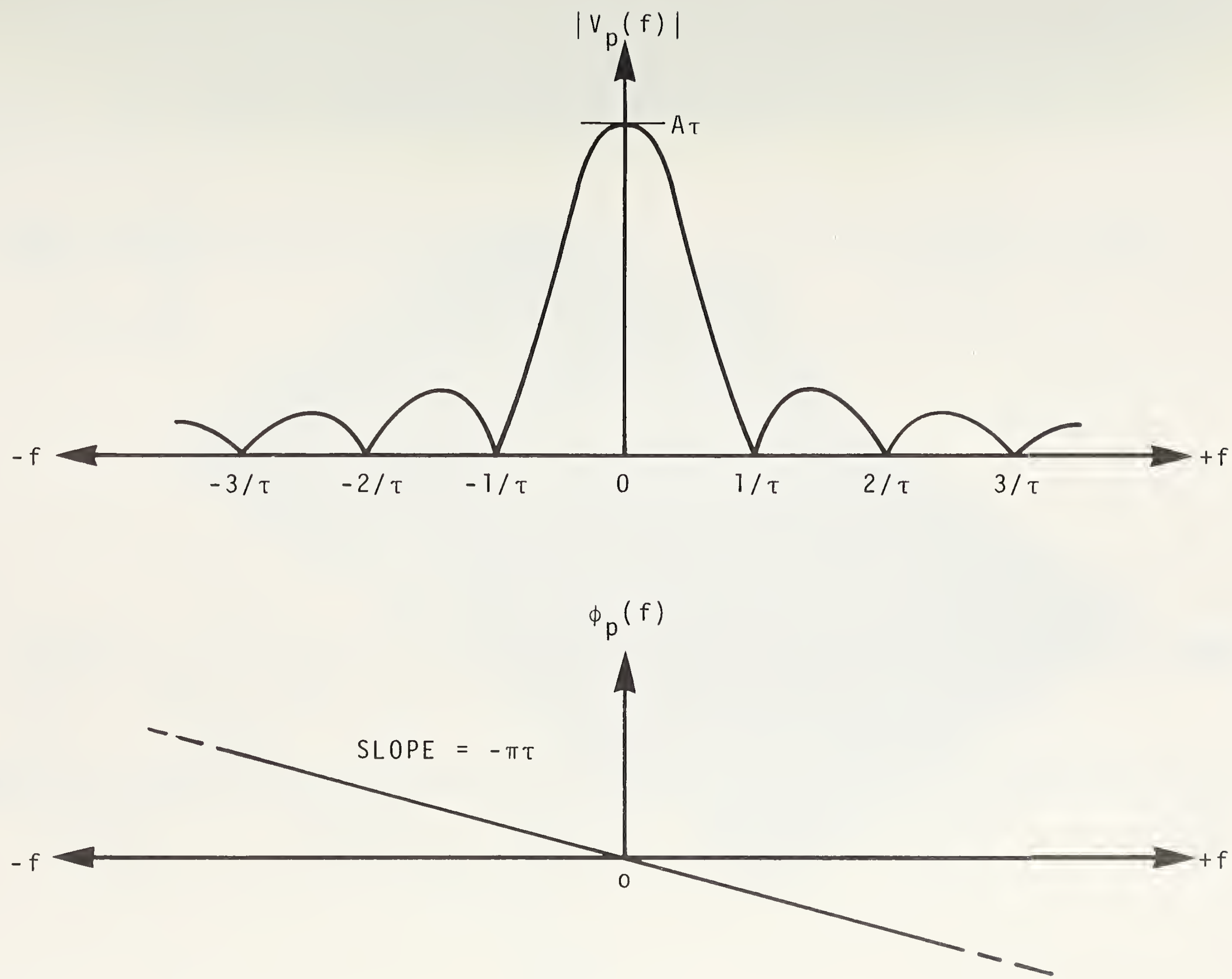

Fig. 4. Plots of $|V_p(f)|$ and $\phi_p(f)$ versus frequency.

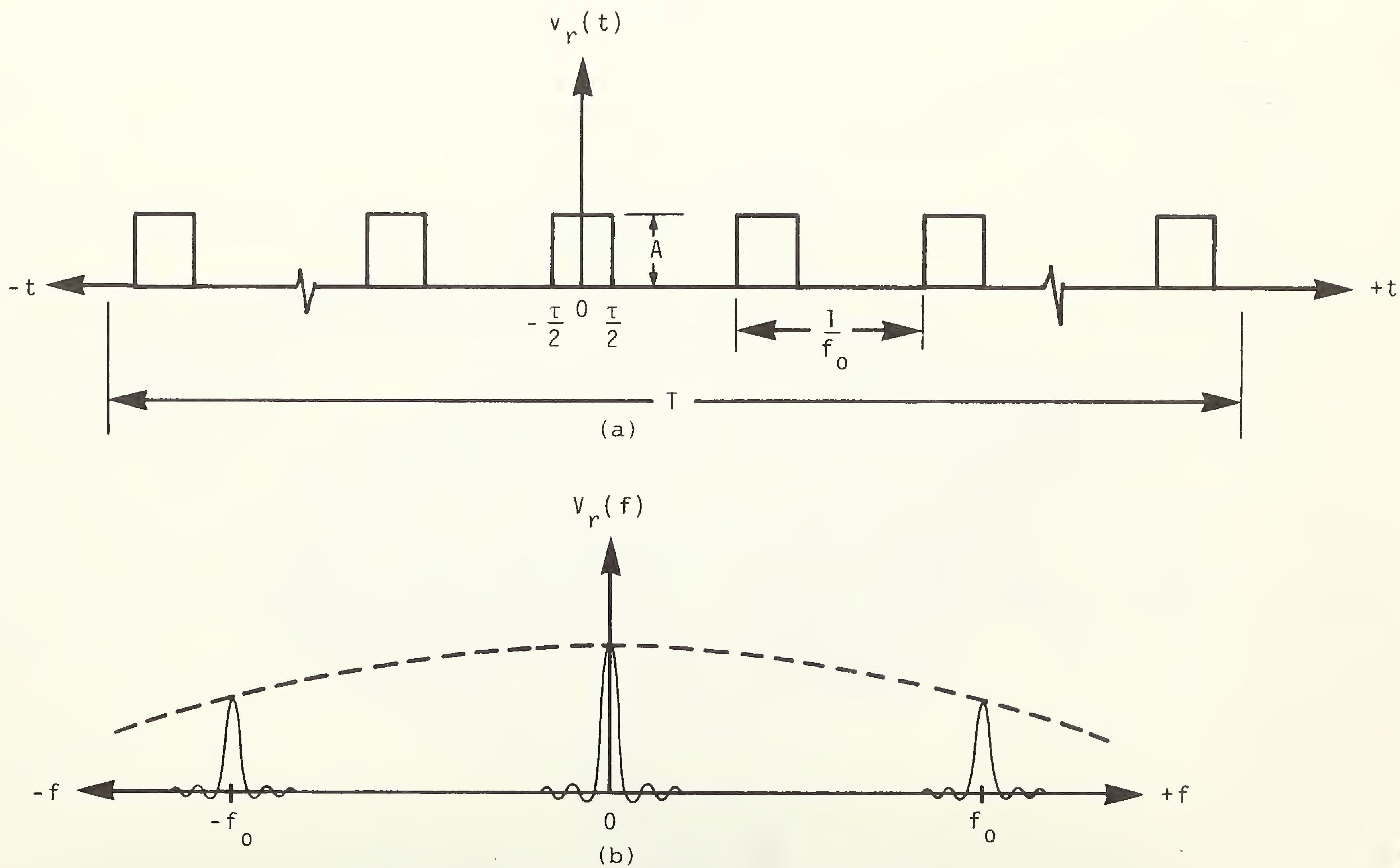

Fig. 5. (a) Finite, regularly repeating rectangular baseband pulse train. (b) Fourier transform of above pulse train.

$$z(f,t) = A\tau \left| \frac{\sin \pi f\tau}{\pi f\tau} \right| \cos 2\pi f(t - \tau/2)$$

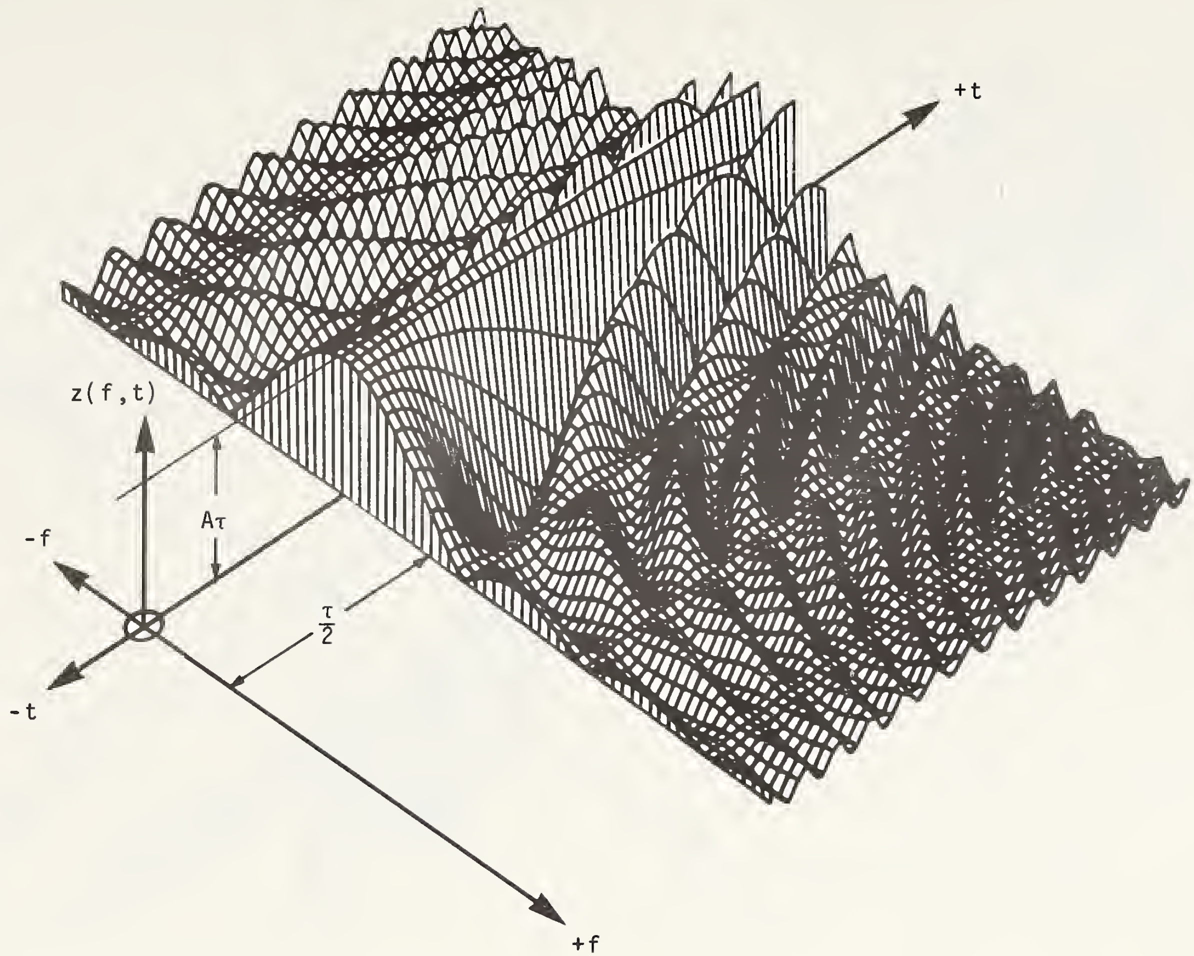

Fig. 6. Three-dimensional plot of z(f,t).

$$z(f,t) = A\tau \left| \frac{\sin \pi f \tau}{\pi f \tau} \right| \cos 2\pi f(t - \tau/2)$$

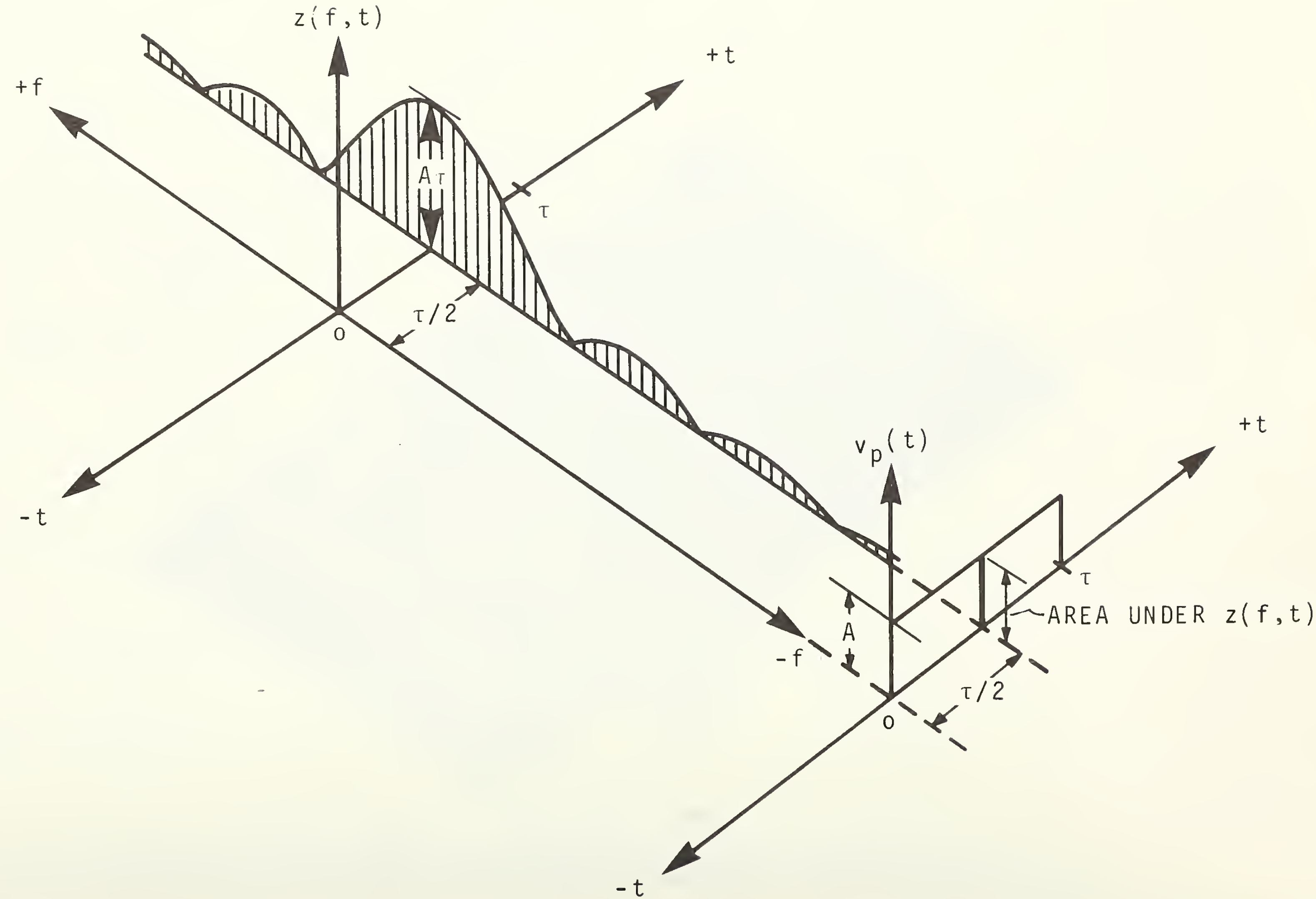

Fig. 7. Illustrating the relationship between $z(f,t)$ and $v_p(t)$.

APPENDIX A

For the baseband pulse, figure 1,

$$v_p(t) = A, \quad 0 < t < \tau \tag{29}$$
$$= 0 \text{ elsewhere.}$$

The Fourier transform, $V_p(f)$, of $v_p(t)$ is

$$V_p(f) = \int_{-\infty}^{\infty} v_p(t) e^{-j2\pi ft}\, dt \tag{8}$$

$$= A \int_{o}^{\tau} e^{-j2\pi ft}\, dt \tag{9}$$

$$= \frac{A}{-j2\pi f}[e^{-j2\pi ft}]_{o}^{\tau}$$

$$= \frac{A}{2\pi f}[j(e^{-j2\pi f\tau} - 1)]$$

$$= \frac{A}{2\pi f}[\sin 2\pi f\tau + j(\cos 2\pi f\tau - 1)]$$

$$= \frac{A}{2\pi f}[\sin 2\pi f\tau - j2 \sin^2 \pi f\tau]. \tag{4}$$

Writing $V_p(f)$ in polar form,

$$V_p(f) = |V_p(f)| e^{j\phi_p(f)} \tag{30}$$

where

$$|V_p(f)| = \left| \frac{A}{2\pi f}[(\sin 2\pi f\tau)^2 + (-2 \sin^2 \pi f\tau)^2]^{1/2} \right|$$

$$= \left| \frac{A}{2\pi f}[\sin^2 2\pi f\tau + 4 \sin^4 \pi f\tau]^{1/2} \right|$$

$$= \left| \frac{A}{2\pi f}[4 \sin^2 \pi f\tau \cos^2 \pi f\tau + 4 \sin^4 \pi f\tau]^{1/2} \right|$$

$$= \left| \frac{A}{\pi f} \sin \pi f\tau [\cos^2 \pi f\tau + \sin^2 \pi f\tau]^{1/2} \right|$$

$$= A\tau \left| \frac{\sin \pi f\tau}{\pi f\tau} \right|. \tag{6}$$

and

$$\phi_p(f) = \tan^{-1}\left\{\frac{-2\ \sin^2\ \pi f\tau}{\sin\ 2\pi f\tau}\right\}$$

$$= \tan^{-1}\left\{\frac{-2\ \sin^2\ \pi f\tau}{2\ \sin\ \pi f\tau\ \cos\ \pi f\tau}\right\}$$

$$= \tan^{-1}(-\tan\ \pi f\tau)$$

$$= -\pi f\tau. \tag{7}$$

Therefore,

$$V_p(f) = A\tau\left|\frac{\sin\ \pi f\tau}{\pi f\tau}\right|e^{-j\pi f\tau}. \tag{5}$$

APPENDIX B

For the finite regularly repeating function shown in figure 5(a), $v_r(t)$ is the convolution of an eternal periodic function, $v_e(t)$, and a gate function, $g(t)$,

$$v_r(t) = v_e(t) * g(t) \tag{31}$$

where

$$\begin{aligned} v_e(t) &= A, \quad -\frac{\tau}{2} < t < \frac{\tau}{2} \\ &= 0, \begin{cases} \frac{\tau}{2} < t < \frac{1}{f_o} - \frac{\tau}{2} \\ -\frac{1}{f_o} + \frac{\tau}{2} < t < -\frac{\tau}{2} \end{cases} \\ &= A, \begin{cases} \frac{1}{f_o} - \frac{\tau}{2} < t < \frac{1}{f_o} + \frac{\tau}{2} \\ -\frac{1}{f_o} - \frac{\tau}{2} < t < -\frac{1}{f_o} + \frac{\tau}{2} \end{cases} \\ &= 0, \begin{cases} \frac{1}{f_o} + \frac{\tau}{2} < t < \frac{2}{f_o} - \frac{\tau}{2} \\ -\frac{2}{f_o} + \frac{\tau}{2} < t < -\frac{1}{f_o} - \frac{\tau}{2} \end{cases} \\ &\text{etc.,} \end{aligned} \tag{32}$$

and

$$\begin{aligned} g(t) &= 1, \quad -\frac{T}{2} < t < \frac{T}{2} \\ &= 0 \text{ elsewhere.} \end{aligned} \tag{33}$$

From the time-convolution theorem,

$$v_e(t) * g(t) \leftrightarrow V_e(f)G(f) \tag{34}$$

where $V_p(f)$ and $G(f)$ are the Fourier transforms of $v_e(t)$ and $g(t)$, respectively. The symbol ($\leftrightarrow$) reads "is the transform of" in both directions.

The Fourier transform of $v_e(t)$ is [6]

$$V_e(f) = A\pi f_o \sum_{n=-\infty}^{n=\infty} \frac{\sin \pi n f_o \tau}{\pi n f_o \tau} \cdot \delta(f - n f_o) \tag{35}$$

where δ is the unit impulse function. The Fourier transform of $g(t)$ is

$$G(f) = T \frac{\sin \pi f T}{\pi f T}, \tag{36}$$

which is similar to eq. (4) except for the zero imaginary part which results from the choice of time origin. Therefore, the Fourier transform of $v_r(t)$ is

$$V_r(f) = V_e(f)G(f)$$

$$= A\tau f_o T \frac{\sin \pi fT}{\pi fT} \sum_{n=-\infty}^{n=\infty} \frac{\sin \pi n f_o \tau}{\pi n f_o \tau} \cdot \delta(f-nf_o)$$

$$= A\tau f_o T \left\{ \sum_{n=-\infty}^{n=\infty} \frac{\sin \pi n f_o \tau}{\pi n f_o \tau} \cdot \frac{\sin \pi(f-nf_o)T}{\pi(f-nf_o)T} \right\}. \tag{14}$$

APPENDIX C

For the baseband pulse, figure 1, we use eqs. (3) and (5) to obtain

$$v_p(t) = \int_{-\infty}^{\infty} V_p(f) e^{j2\pi ft}\, df$$

$$= A\tau \int_{-\infty}^{\infty} \left|\frac{\sin \pi f\tau}{\pi f\tau}\right| e^{j(2\pi ft - \pi f\tau)}\, df$$

$$= A\tau \int_{-\infty}^{\infty} \left|\frac{\sin \pi f\tau}{\pi f\tau}\right| [\cos 2\pi f(t-\tau/2) - j \sin 2\pi f(t-\tau/2)] df$$

$$= A\tau \int_{-\infty}^{\infty} \left|\frac{\sin \pi f\tau}{\pi f\tau}\right| \cos 2\pi f(t-\tau/2) df \qquad (15)$$

since

$$\int_{-\infty}^{\infty} \left|\frac{\sin \pi f\tau}{\pi f\tau}\right| \sin 2\pi f(t-\tau/2) df = 0.$$

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The term, impulse spectral intensity, is often used in discussions concerning electromagnetic interference and broadband signal processing. In these discussions, the term is not used in a consistent manner, resulting in confusion, equivocation, and sometimes, errors. As a step towards improving this situation, the mathematical basis for spectral intensity is reviewed and certain of its features are clarified. Also, misuses of spectral intensity are discussed, along with its limitations and proper use.

17. KEY WORDS (Alphabetical order, separated by semicolons) Electromagnetic interference; Fourier transform; impulse spectral intensity; spectral intensity; spectrum amplitude; spectrum amplitude density.

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